

Exam 08.09.2026

Instruction: Answer **ALL** questions.**Allowed:** Writing tools, a calculator (any model), and either one mathematics table book OR one one-sided A4-sized cheat sheet.

1. a. The logic function F has inputs A , B , C and D and fulfils the truth table below. Use a Karnaugh map to obtain the simplest SOP form of F . Draw the corresponding simplified logic circuit. Show your Karnaugh-map groups clearly. (3 pts)

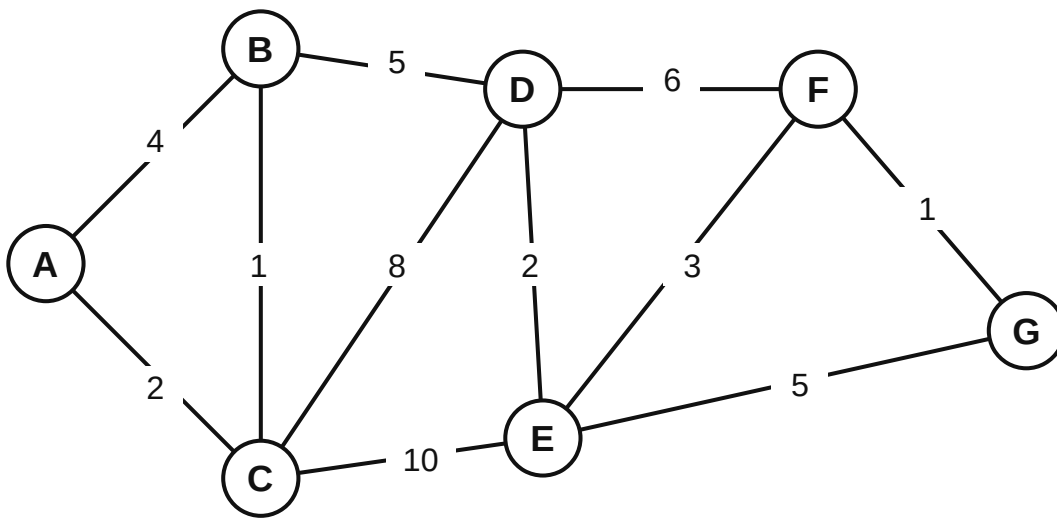
A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

b. A laboratory uses the following reasoning:

If the alarm is active, the warning light is on. If the warning light is not on, the emergency door remains locked. The emergency door is not locked. Therefore, the alarm is not active.

Let a = "the alarm is active", w = "the warning light is on", and d = "the emergency door is locked". Formalize the premises and conclusion, and determine whether the argument is valid. Justify your answer. (2 pts)

2. The following undirected weighted graph represents travel times between locations. Use Dijkstra's algorithm to determine the shortest paths from vertex A to every other vertex.



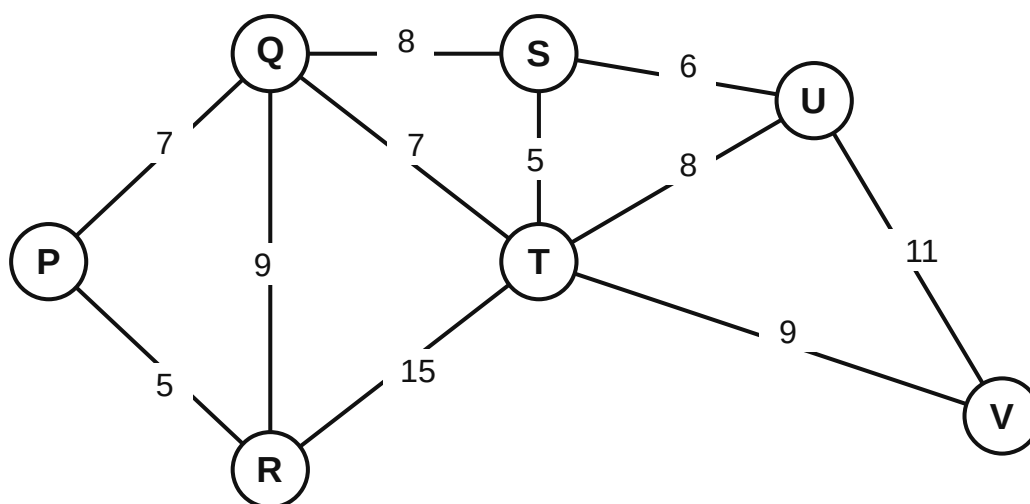
Required:

- Construct the weighted adjacency matrix. Order its rows and columns alphabetically. (1 pt)
- Present a complete Dijkstra table. (2.5 pts)
- State the final shortest distance and corresponding path from A to each other vertex. (1.5 pts)

3. Are the following claims true (T) or false (F)? Correct answer = 1 point; incorrect answer = -0.5 points; no answer = 0 points. Justifications are welcome but not required. The total for this question cannot be negative. (5 pts)

- $\neg(p \wedge q)$ is logically equivalent to $\neg p \wedge \neg q$.
- 347 and 25 are congruent modulo 14.
- A connected graph with exactly two vertices of odd degree has an Euler circuit.
- Every symmetric and transitive relation on a nonempty set is necessarily reflexive.
- The sequence $y_n = 3 \cdot 2^n$ satisfies $y_{n+1} - 2y_n = 0$.

4. The vertices below represent facilities that must all be connected at minimum total cost. Starting from vertex P , use Prim's algorithm to find a minimum spanning tree. (5 pts)



Required:

- Apply Prim's algorithm correctly and clearly list or draw the selected minimum-spanning-tree edges. (2 pts)
- Present the weighted adjacency matrix and a complete Prim table. At each iteration show the vertices already in the tree, the candidate connecting edges, and the selected edge. (2 pts)
- Calculate the sum of the weights in the completed tree. (1 pt)

5. An undirected simple graph G has vertices $\{A, B, C, D, E\}$ and edge set

$$E(G) = \{AB, AC, BC, BD, CD, CE, DE\}.$$

Order all rows and columns alphabetically.

- Construct the adjacency matrix A . (1 pt)
- Construct the degree matrix D . (1 pt)
- Calculate the Laplace matrix. (2 pts)
- Use your Laplace matrix to state the degree of vertex C , verify that every row sum is zero, and briefly explain what an off-diagonal entry $l_{ij} = -1$ means. (1 pt)