

BM20A8000 Differential Equations

Exam 23.9.2026

Only the following written materials are allowed in the exam: MAOL, BETA, Tekniikan taulukkokirja (Valtanen), Matematiikan taulukkokirja (Valtanen), and a handwritten A4-sized formula sheet. The formula sheet must be returned with the answer sheets.

Calculators are not allowed.

Answer each question.

1. (a) Solve the initial value problem

$$\left(\frac{x}{y}\right)^2 - y' = 0, \quad y(0) = 1. \quad (3p)$$

- (b) Solve the initial value problem

$$y'' + y' = 0, \quad y(0) = 0, \quad y'(0) = 2. \quad (3p)$$

2. (a) Solve the initial value problem

$$y' - 2y - 4 = 0, \quad y(0) = 1. \quad (3p)$$

- (b) Write the initial value problem

$$xy^{(3)} - 3y'' + x^2y' - 5y + 1 = 0, \quad y(0) = -2, \quad y'(0) = 1, \quad y''(0) = 0,$$

as a system of first-order differential equations and provide the corresponding initial conditions. (3p)

3. Solve the initial value problem

$$y'' - y - x = 0, \quad y(0) = 1, \quad y'(0) = 1. \quad (6p)$$

4. (a) Write the complex number $z = 1 - \sqrt{3}i$ in exponential form $re^{\theta i}$ for some $r \geq 0$ and $\theta \in \mathbb{R}$. (3p)

- (b) Show that $z = e^{-\frac{\pi}{2}i}$ is one solution of the equation $z^3 - 2z^2 + z - 2 = 0$. (3p)

5. (a) Determine the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} -2 & 5 \\ 4 & -1 \end{bmatrix}. \quad (3p)$$

- (b) Solve the initial value problem

$$\mathbf{y}' = A\mathbf{y}, \quad \mathbf{y}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

where A is the matrix from part (a). (3p)