Final Examination

Course: LES10A210 Engineering Mathematics II
Examiner: Barkat Bhayo
Date: 06.05.2026 Total marks: 20

Note: Use of calculator is not allowed. Include the applied formulas, substitutions, and intermediate steps in your solution of attempting question. Submitting only the final value/answer is not acceptable.

Attempt any five questions, all questions carry equal marks.
In whole paper z is a complex number and $i = \sqrt{-1}$

Question #1.

(a) The curve is given by the following parametric equations

$$x = \sec t \quad y = \tan t, \quad -\frac{\pi}{2} < t < \frac{\pi}{2}.$$

Find the equation of the tangent line to the curve at the point, where $t = \pi/4$.

(b) Find the length of the curve parametrized by

$$x = 2t - \sin(2t), \quad y = 5\sin^2 t, \ 0 \le t \le \pi.$$

Hint : length s of a curve is given by the fomula

$$s = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

Question #2.

(a) Find

$$\lim_{z \to 1+i} \frac{z^2 - 2i}{z^2 - 2z + 2}$$

(b) Find the spherical equation for the surface whose rectangular equation is

$$y = xz\,.$$

Question #3.

(a) Write the complex number $z = -\sqrt{3} - 1$ in the form

$$z = r(\cos\theta + i\sin\theta)\,.$$

(b) Find $A^2 - 5A + 4I_3$, if $A = \begin{pmatrix} -2 & 1 & 0 \\ -1 & 4 & 3 \\ 0 & 8 & 5 \end{pmatrix}$, where I_3 is a unit matrix of order 3×3.

Question #4.

(a) Determine the characteristic equation and eigenvalues λ in the system

$$\begin{pmatrix} 3 & 1 \\ -1 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \end{pmatrix}.$$

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(b) Show that the vectors $\vec{a} = \vec{i} + 3\vec{j} + 5\vec{k}$, $\vec{b} = -2\vec{i} + 4\vec{j} - 6\vec{k}$ and $\vec{c} = -3\vec{i} + \vec{j} - 11\vec{k}$ are coplanar.

Question #5.

(a) Compute

$$(1+i)^{24}\,.$$

(b) Write $f(z) = z^5 + 4z^2 - 6$ in the form $f(z) = u(r,\, \theta) + iv(r,\, \theta)$.

Question #6.

(a) Find the rectangular coordinates of the point P, whose spherical coordinates are $(4,\, \pi/6,\, \pi/3)$.

(b) Write $f(z) = z^4$ in the form $f(z) = f(x+iy) = u(x,\, y) + iv(x,\, y)$.

Question #7.

(a) Write $\sqrt{3}\, e^{i\frac{3\pi}{4}}$ in the form $x + iy$.

(b) Prove that $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}\,$.