BM20A8601 Statistics I
Tentti 06.05.2026

You may use a calculator in the exam. Attachments: normal and t-tables and a collection of formulas.
Remember to write down your reasoning or steps to the solution.

Task 1

You ask your neighbour to water a sickly plant while you are on vacation. Without water, it will die with probability 0.8; with water, it will die with probability 0.15. You are 90 per cent certain that your neighbour will remember to water the plant.

(a) What is the probability that the plant will be alive when you return?

(b) If it is dead, what is the probability your neighbour forgot to water it?

Task 2

In the following tasks, in addition to solving the task give reasoning for why the given distribution is an appropriate one for describing the events. In the case no distribution is given, deduce what you should use and why.

(a) Let X be the number of cars per minute passing a certain point of some road between 8 A.M. and 10 A.M. on a Sunday. Assume that X has a Poisson distribution with mean 5. Find the probability of observing 4 or fewer cars during any given minute.

(b) The number of years a radio functions is exponentially distributed with parameter $\lambda = 18$. If Jorma buys a used radio, what is the probability that it will be working after an additional 10 years?

(c) The multiple-choice exam has 10 questions, each with 4 answer choices, of which one is correct. To pass the test, the student must answer at least 8 questions correctly. Calculate the probability of a student passing if they choose all the answers at random.

Task 3

The range of a new type of mortar shell is being investigated. The observed ranges, in meters, of 20 such shells are as follows: 2,100 1,984 2,072 1,898 1,950 1,992 2,096 2,103 2,043 2,218 2,244 2,206 2,210 2,152 1,962 2,007 2,018 2,106 1,938 1,956.

Assuming that a shell's range is normally distributed, construct **(a)** a 95 per cent and **(b)** a 99 per cent two-sided confidence interval for the mean range of a shell.

Task 4

A colony of laboratory mice consists of several thousand mice. The average weight of all the mice is 32 grams with a standard deviation of 4 grams. A scientist asked a laboratory assistant to select 25 mice for an experiment. However, before performing the experiment, the scientist decided to weigh the mice as an indicator of whether the assistant's selection constituted a random sample or whether it was made with some unconscious bias (perhaps the mice selected were the ones that were slowest in avoiding the assistant, which might indicate some inferiority about this group). If the sample mean of the 25 mice was 30.4, would this be significant evidence, at the 5 per cent level of significance, against the hypothesis that the selection constituted a random sample?

Task 5

A certain component is critical to the operation of an electrical system and must be replaced immediately upon failure. If the mean lifetime of this type of component is 100 hours and its standard deviation is 30 hours, how many of the components must be in stock so that the probability that the system is in continual operation for the next 2000 hours is at least 0.95?

Standard normal table

The table below contains approximate values of the cumulative distribution function of the standard normal distribution

$$\Phi(x) = F_Z(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt.$$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

Student's t-distribution table

The table below contains approximate values of the quantile function

$$F_{t,n-1}^{-1}(p)$$

where $F_{t,n-1}$ is the cumulative distribution function for Student's t-distribution with degrees of freedom $n-1$. For example, $P(T \leq 1.2497\ldots) = p$ when $n-1=3$, $p=0.85$ and T has the t-distribution with degrees of freedom $n-1$.

	p									
$n-1$	0.5000	0.7500	0.8000	0.8500	0.9000	0.9500	0.9750	0.9900	0.9950	0.9990
1	0.0000	1.0000	1.3764	1.9626	3.0777	6.3138	12.7062	31.8205	63.6567	318.3088
2	0.0000	0.8165	1.0607	1.3862	1.8856	2.9200	4.3027	6.9646	9.9248	22.3271
3	0.0000	0.7649	0.9785	1.2498	1.6377	2.3534	3.1824	4.5407	5.8409	10.2145
4	0.0000	0.7407	0.9410	1.1896	1.5332	2.1318	2.7764	3.7469	4.6041	7.1732
5	0.0000	0.7267	0.9195	1.1558	1.4759	2.0150	2.5706	3.3649	4.0321	5.8934
6	0.0000	0.7176	0.9057	1.1342	1.4398	1.9432	2.4469	3.1427	3.7074	5.2076
7	0.0000	0.7111	0.8960	1.1192	1.4149	1.8946	2.3646	2.9980	3.4995	4.7853
8	0.0000	0.7064	0.8889	1.1081	1.3968	1.8595	2.3060	2.8965	3.3554	4.5008
9	0.0000	0.7027	0.8834	1.0997	1.3830	1.8331	2.2622	2.8214	3.2498	4.2968
10	0.0000	0.6998	0.8791	1.0931	1.3722	1.8125	2.2281	2.7638	3.1693	4.1437
11	0.0000	0.6974	0.8755	1.0877	1.3634	1.7959	2.2010	2.7181	3.1058	4.0247
12	0.0000	0.6955	0.8726	1.0832	1.3562	1.7823	2.1788	2.6810	3.0545	3.9296
13	0.0000	0.6938	0.8702	1.0795	1.3502	1.7709	2.1604	2.6503	3.0123	3.8520
14	0.0000	0.6924	0.8681	1.0763	1.3450	1.7613	2.1448	2.6245	2.9768	3.7874
15	0.0000	0.6912	0.8662	1.0735	1.3406	1.7531	2.1314	2.6025	2.9467	3.7328
16	0.0000	0.6901	0.8647	1.0711	1.3368	1.7459	2.1199	2.5835	2.9208	3.6862
17	0.0000	0.6892	0.8633	1.0690	1.3334	1.7396	2.1098	2.5669	2.8982	3.6458
18	0.0000	0.6884	0.8620	1.0672	1.3304	1.7341	2.1009	2.5524	2.8784	3.6105
19	0.0000	0.6876	0.8610	1.0655	1.3277	1.7291	2.0930	2.5395	2.8609	3.5794
20	0.0000	0.6870	0.8600	1.0640	1.3253	1.7247	2.0860	2.5280	2.8453	3.5518
21	0.0000	0.6864	0.8591	1.0627	1.3232	1.7207	2.0796	2.5176	2.8314	3.5272
22	0.0000	0.6858	0.8583	1.0614	1.3212	1.7171	2.0739	2.5083	2.8188	3.5050
23	0.0000	0.6853	0.8575	1.0603	1.3195	1.7139	2.0687	2.4999	2.8073	3.4850
24	0.0000	0.6848	0.8569	1.0593	1.3178	1.7109	2.0639	2.4922	2.7969	3.4668
25	0.0000	0.6844	0.8562	1.0584	1.3163	1.7081	2.0595	2.4851	2.7874	3.4502
26	0.0000	0.6840	0.8557	1.0575	1.3150	1.7056	2.0555	2.4786	2.7787	3.4350
27	0.0000	0.6837	0.8551	1.0567	1.3137	1.7033	2.0518	2.4727	2.7707	3.4210
28	0.0000	0.6834	0.8546	1.0560	1.3125	1.7011	2.0484	2.4671	2.7633	3.4082
29	0.0000	0.6830	0.8542	1.0553	1.3114	1.6991	2.0452	2.4620	2.7564	3.3962
30	0.0000	0.6828	0.8538	1.0547	1.3104	1.6973	2.0423	2.4573	2.7500	3.3852
31	0.0000	0.6825	0.8534	1.0541	1.3095	1.6955	2.0395	2.4528	2.7440	3.3749
32	0.0000	0.6822	0.8530	1.0535	1.3086	1.6939	2.0369	2.4487	2.7385	3.3653
33	0.0000	0.6820	0.8526	1.0530	1.3077	1.6924	2.0345	2.4448	2.7333	3.3563
34	0.0000	0.6818	0.8523	1.0525	1.3070	1.6909	2.0322	2.4411	2.7284	3.3479
35	0.0000	0.6816	0.8520	1.0520	1.3062	1.6896	2.0301	2.4377	2.7238	3.3400
36	0.0000	0.6814	0.8517	1.0516	1.3055	1.6883	2.0281	2.4345	2.7195	3.3326
37	0.0000	0.6812	0.8514	1.0512	1.3049	1.6871	2.0262	2.4314	2.7154	3.3256

Probability and Statistics Formula Sheet

Probability & Statistics — Cheat Sheet

Probability rules

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A \cup B) = P(A) + P(B), \quad A \cap B = \varnothing$$
$$P(A^c) = 1 - P(A)$$
$$P(A) \le P(B), \quad \text{when } A \subseteq B$$
$$P(A \setminus B) = P(A) - P(A \cap B)$$

Conditional probability & Bayes

$$P(E \mid A) = \frac{P(E \cap A)}{P(A)}, \quad P(A) \neq 0$$
$$P(A) = \sum_{i=1}^{n} P(A \mid B_i)\, P(B_i)$$
$$P(B \mid A) = \frac{P(A \mid B)\, P(B)}{P(A)}$$

Independence: $P(A \cap B) = P(A)\, P(B)$

Expectation and variance

Expectation (discrete / continuous):

$$E(X) = \sum_{\nu} \nu \cdot P(X = \nu)$$
$$E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

Function of two variables:

$$E\big(g(L,S)\big) = \sum_{l} \sum_{s} g(l,s)\, f(l,s)$$

Variance (definition):

$$\mathrm{Var}(X) = E\big[(X-\mu)^2\big]$$

Variance (computational formula):

$$\mathrm{Var}(X) = E(X^2) - \big[E(X)\big]^2$$

Standard deviation: $\mathrm{SD}(X) = \sqrt{\mathrm{Var}(X)}$

Linearity rules:

$$E(aX+b) = a\, E(X) + b$$
$$\mathrm{Var}(aX+b) = a^2\, \mathrm{Var}(X)$$
$$E(X+Y) = E(X) + E(Y)$$

If X, Y are independent:

$$\mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$$

Covariance and correlation

Discrete:

$$\mathrm{Cov}(X,Y) = \sum_{x} \sum_{y} (x-\mu_X)(y-\mu_Y)\, f(x,y)$$

Continuous:

$$\mathrm{Cov}(X,Y) = \iint (x-\mu_X)(y-\mu_Y)\, f(x,y)\, dx\, dy$$

Computational formula:

$$\mathrm{Cov}(X,Y) = E(XY) - E(X)\, E(Y)$$

Sample correlation:

$$r(\vec{x},\vec{y}) = \frac{\sum_j (x_j-\bar{x})(y_j-\bar{y})}{\sqrt{\sum_j (x_j-\bar{x})^2 \; \sum_j (y_j-\bar{y})^2}}$$

Marginal distributions

Discrete random variables X, Y with joint pmf $p_{X,Y}(x,y)$:

$$p_X(x) = \sum_{y} p_{X,Y}(x,y)$$
$$p_Y(y) = \sum_{x} p_{X,Y}(x,y)$$

Distributions

Continuous uniform $X \sim \text{Uniform}(a,b)$

$$f(t) = \begin{cases} \frac{1}{b-a}, & a < t < b \\ 0, & \text{otherwise} \end{cases} \qquad F(t) = \begin{cases} 0, & t < a \\ \frac{t-a}{b-a}, & a \le t \le b \\ 1, & t > b \end{cases}$$

$$E(X) = \tfrac{a+b}{2}, \qquad \text{Var}(X) = \tfrac{(b-a)^2}{12}$$

Exponential $X \sim \text{Exp}(\lambda)$

$$f(t) = \begin{cases} \lambda e^{-\lambda t}, & t > 0 \\ 0, & t \le 0 \end{cases} \qquad F(t) = \begin{cases} 0, & t \le 0 \\ 1 - e^{-\lambda t}, & t > 0 \end{cases}$$

$$E(X) = \tfrac{1}{\lambda}, \qquad \text{Var}(X) = \tfrac{1}{\lambda^2}$$

Normal $X \sim N(\mu, \sigma^2)$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\Big(-\frac{(x-\mu)^2}{2\sigma^2}\Big)$$

$$E(X) = \mu, \qquad \text{Var}(X) = \sigma^2$$

Bernoulli $X \sim \text{Ber}(p)$

$$f(k) = \begin{cases} 1-p, & k = 0 \\ p, & k = 1 \end{cases} \qquad E(X) = p,\ \text{Var}(X) = p(1-p)$$

Binomial $X \sim \text{Bin}(n,p)$

$$f(k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad k = 0, 1, \ldots, n$$

$$E(X) = np, \qquad \text{Var}(X) = np(1-p)$$

Geometric $X \sim \text{Geom}(p)$

$$f(k) = (1-p)^k p, \quad F(k) = 1 - (1-p)^{k+1}$$

$$E(X) = \tfrac{1}{p}, \qquad \text{Var}(X) = \tfrac{1-p}{p^2}$$

Poisson $X \sim \text{Poi}(\lambda)$

$$f(k) = \frac{\lambda^k e^{-\lambda}}{k!}, \quad k = 0, 1, 2, \ldots$$

$$E(X) = \lambda, \qquad \text{Var}(X) = \lambda$$

Z-score and confidence interval

Standardization:

$$Z = \frac{X - \mu}{\sigma} \sim N(0,1)$$

Standardized statistic (σ known):

$$NV(\vec{X}) = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

t statistic (σ unknown):

$$\frac{\bar{X} - \mu}{s/\sqrt{n}} \sim t_{n-1}$$

Confidence interval at level α:

$$\bar{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \quad \text{or} \quad \bar{X} \pm t_{\alpha/2,\, n-1} \frac{s}{\sqrt{n}}$$

Sample quantities

Sample mean:

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

Sample variance (unbiased):

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$$

Sample standard deviation: $s = \sqrt{s^2}$

Central Limit Theorem (CLT)

If $X_1, \ldots, X_n$ are i.i.d. with $E(X_i) = \mu$, $\text{Var}(X_i) = \sigma^2 < \infty$:

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \xrightarrow{d} N(0,1), \quad n \to \infty$$

Useful rules

Complement rule: $P(A^c) = 1 - P(A)$

De Morgan's laws:

$$(A \cup B)^c = A^c \cap B^c$$
$$(A \cap B)^c = A^c \cup B^c$$