BM20A8401 Partial Differential Equations
Exam 06.05.2026

Exam Instructions

- Only stationery (pen and empty paper) is allowed in the exam. *(Sama Suomeksi: Ainoastaan tavalliset kirjoitusvälineet ovat sallittuja tentissä.)*
- Answer to **all five problems**; each is worth **6 marks**; the total marks for the exam is **30**.

1. **Instructions:**
   - For each statement below, determine whether it is **True** or **False**.
   - You will receive **1 mark** for each correct answer, **-1 mark** for each incorrect answer and **0 marks** for each unanswered statement.
   - However, the total marks for this question cannot be negative, meaning that even if you accumulate negative points, the final score for this question will be at least **0 marks**.

   Statements:

   (i) Every sequence in $[0,1]$ has a subsequence that converges to a point in $[0,1]$.

   (ii) The function $f : [0,1] \to \mathbb{R}$ defined by $f(x) = x^{1/3}$ is uniformly continuous on $[0,1]$, but it is not Lipschitz continuous on $[0,1]$.

   (iii) Let $\alpha = (2,3,2) \in \mathbb{N}_0^3$ be a multi-index. Then $|\alpha| = 7$ and $\alpha! = 12$.

   (iv) The function $u : \mathbb{R}^2 \setminus \{(0,0)\} \to \mathbb{R}$, given by $u(x,y) = \log|x^2 + y^2|$ is harmonic.

   (v) The set $\mathbb{Q}$ of rational numbers is complete as a subset of $\mathbb{R}$ with the usual topology.

   (vi) Let $\phi : \mathbb{R}^n \to \mathbb{R}$ be continuous. If $\phi(x) = 0$ outside some bounded set then the support of ϕ is compact.

2. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by $f(x,y) = e^{2x}\cos y$. Determine the Taylor polynomial of degree 2 of f centered at $(1,\pi)$.

3. (a) Let
$$\Omega = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 < 4\}.$$
   Suppose $u \in C^2(\Omega) \cap C(\overline{\Omega})$ solves
$$\begin{cases} u_{xx} + u_{yy} = 0 & \text{in } \Omega, \\ u(x,y) = 2 - 3x + 4y & \text{on } \partial\Omega. \end{cases}$$
   Find the value of $u(1,-1)$.

   (b) Let
$$\Omega = \{(x,y) \in \mathbb{R}^2 :\ x^2 + y^2 < 1\}.$$
   Assume $u \in C^2(\Omega) \cap C(\overline{\Omega})$ is a solution of
$$\begin{cases} \Delta u = 0 & \text{in } \Omega, \\ u = 4 + x^2 + y^2 + 6xy & \text{on } \partial\Omega. \end{cases}$$
   Determine the maximal value of u in Ω, i.e.,
$$\max_{\overline{\Omega}} u.$$

4. Find a solution to the first-order partial differential equation

$$xu_x + yu_y = 2$$

in the set

$$\Omega = \left\{(x, y) \in \mathbb{R}^2 : x > 0\right\},$$

satisfying the condition

$$u(1, y) = 3y^2.$$

5. (a) Define what it means for a function f to be smooth.
   (b) Define what it means for a function f to be real analytic.
   (c) Let $\Omega \subset \mathbb{R}^n$ be an open connected set. Investigate whether the following statements are true or false (justify your answer with a proof or a counterexample):
      (i) If f is smooth in Ω, then f is harmonic in Ω.
      (ii) If f is real analytic in Ω, then f is harmonic Ω.