

Exam 3, 2025

Date 08,05,2026

*Duration – 3 Hours**Total Marks – 100**Note: (a) Calculators are allowed in the exam.**(b) A formula sheet is provided at the end of the exam paper for reference according to the question number.*

1. A non-linear pressure sensor has an input range of 0 to 1000 kPa and an output range of 0 to 50 mV. When the input pressure is 300 kPa, the sensor outputs 12 mV. Additionally, the maximum non-linearity occurs at 500 kPa, where the ideal output would be 25 mV, but the actual output is 23.5 mV.

**Calculate:**

- (a) The non-linearity at 300 kPa in millivolts. [3 Marks]  
 (b) The non-linearity at 300 kPa as a % of the output span. [3 Marks]  
 (c) The maximum non-linearity as a % of the output span. [4 Marks]

2. A student measures the diameter of a metal rod using a micrometre. She takes 10 repeated measurements (in mm):

15.42, 15.47, 15.45, 15.44, 16.02, 15.46, 15.43, 15.45, 15.48, 15.44

**Calculate:**

- (a) The mean diameter of a metal rod. [5 Marks]  
 (b) The experimental standard deviation of the measurements. [5 Marks]

3. A set of calibration data from a temperature sensor is given below:

$T$ (°C)	0	5	10	15	20	25	30	35
$V$ (mV)	1.0	1.7	2.5	3.0	3.8	4.4	5.3	5.9

The sensor output should ideally follow a straight line of the form:

$$V = mT + c$$

**Tasks:**

- (a) Use the least squares method to determine  $m$  and  $c$ . [6 marks]  
 (b) Predict the sensor output when  $T = 40^\circ\text{C}$  using your fitted model. [2 marks]  
 (c) Compute the residual at  $T = 25^\circ\text{C}$  (difference between measured and predicted output). [2 marks]

4. Generate a linearized model about the given operating point using a first-order Taylor series approximation.

(a)  $h(x) = 5x^3 - 2x^2 + 6x - 3$  at  $x = 1$  [5 marks]

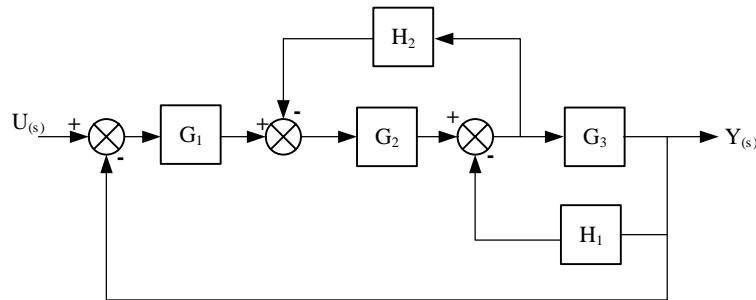
(b)  $f(x) = 3\sin(2x) + \frac{5}{x-1}$  at  $x = 2$ . Given  $\sin(2x)$  is in radians [5 marks]

5. The open loop transfer function of the unity feedback system is

$$\frac{K}{s(1 + 0.5s)(1 + 0.2s)}$$

Find the restrictions of  $K$ , so that the closed-loop system is stable. [10 marks]

6. Determine the ratio  $\frac{Y(s)}{U(s)}$  of the system shown in the figure. [10 marks]



7. The closed-loop transfer function of a control system is

$$\frac{C(s)}{R(s)} = \frac{16}{s^3 + 10s^2 + 31s + 30}$$

**Tasks:**

- (a) Find the real-time response  $c(t)$  when the input is a unit step  $R(s) = 1/s$ . [7 Marks]  
 (b) Identify the steady-state value of  $c(t)$ . [3 Marks]

8. A system is described by:

$$\dot{x}(t) = \begin{bmatrix} 0 & 2 \\ -3 & -4 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = [1 \quad 0]x(t)$$

**Tasks:**

- (a) Draw a simulation block diagram from the state equations. [5 marks]  
 (b) Determine the transfer function  $\frac{Y(s)}{U(s)}$  using the state-space formula. [3 marks]  
 (c) If you replace the output matrix  $C$  with  $[0 \quad 1]$ , what is the new transfer function? [2 marks]

9. The forward path transfer function of a unity feedback control system is:

$$G(s) = \frac{K(s + 5)}{s(s + 2)(s^2 + 3s + 10)}$$

where  $K > 0$ .

**Tasks:**

- (a) Determine the type of the system. [2 marks]  
 (b) Find the position, velocity, and acceleration error coefficients in terms of  $K$ . [6 marks]  
 (c) For  $K = 20$ , compute numerical values of coefficients. [2 marks]

10. A system's transfer function is:

$$G(s) = \frac{s + 5}{(s^2 + 9)(s + 2)(s^2 + 2s + 5)}$$

- (a) Find the poles and zeros of the system. [5 marks]  
 (b) Determine if the system is stable or not using the pole-zero position in the s-plane. [5 marks]

**Formula sheet**

(1) Nonlinearity  $N(I) = O(I) - (K \cdot I + a)$

$$\% \text{ Nonlinearity} = \frac{\text{Nonlinearity}}{\text{Output span}} \times 100 \%$$

(2) Estimated standard deviation  $s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{(n-1)}}$

Mean (average):  $\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$

(3) Least square (LS) method for straight-line:

- $y = Kx + a$

- $K = \frac{n \sum I_i O_i - \sum I_i \sum O_i}{n \sum I_i^2 - (\sum I_i)^2}$

- $a = \frac{\sum O_i - K \sum I_i}{n}$

(4) Taylor's series approximation  $Y(x) = f(p) + \frac{f'(p)}{1!} (x - p)$

(8) The transfer function  $G(s)$  is given by:  $G(s) = C(sI - A)^{-1}B + D$

(9) Position error constant  $K_p = \lim_{s \rightarrow 0} G(s)H(s)$

Velocity error constant  $K_v = \lim_{s \rightarrow 0} sG(s)H(s)$

Acceleration error constant  $K_a = \lim_{s \rightarrow 0} s^2 G(s)H(s)$