# BM20A8101 Integral Calculus
# Exam 28.4.2026

Only the following written materials are allowed in the exam: MAOL, BETA, Tekniikan taulukkokirja (Valtanen), Matematiikan taulukkokirja (Valtanen), and a handwritten A4-sized formula sheet.

Calculators are not allowed.

Answer each question.

Integrals must be evaluated completely unless stated otherwise.

1. (a) Calculate
$$\int_{-1}^{1} |x^2 - x| \, \mathrm{d}x. \tag{3p}$$
(b) Calculate
$$\int_{1}^{2} x\sqrt{x-1} \, \mathrm{d}x$$
by using the substitution $x = t + 1$. (3p)

2. (a) The function $f(x) = \sqrt{x}$ and the lines $y = 0$ and $x = 1$ enclose a finite area, which is rotated around the $x$-axis. Calculate the volume of the resulting solid of revolution. (3p)
(b) A four-meter-long thin metal rod lies along the $x$-axis in the interval $x \in [0, 4]$. Its density is
$$\rho(x) = 1 + \sqrt{x}, \qquad x \in [0, 4],$$
where $x$ is measured in meters. Find the $x$-coordinate of the rod's center of mass. (3p)

3. (a) Compute the area bounded by the $x$-axis and the parametric curve
$$x(t) = t^2, \quad y(t) = t^3 \quad \text{for } t \in [0, 2]. \tag{3p}$$
(b) A robot travels along a path defined by the parametric curve
$$x(t) = t\cos(t), \quad y(t) = t\sin(t).$$
Determine the length of the path in the interval $t \in [0, 3]$. *You do not need to evaluate the resulting integral in part (b), but you should simplify the integrand to the simplest possible form.* (3p)

4. The lifetime $T$ of a radioactive particle has the probability density function
$$f(t) = \begin{cases} e^{-t} & \text{if } t \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$
The probability that the lifetime $T$ is in the interval $[a, b]$ is given by $P(a \leq T \leq b) = \int_a^b f(t) \, \mathrm{d}t$.
(a) Compute the probability $P(0 \leq T \leq 1)$. (3p)
(b) What is the half-life of this particle (i.e. the time $m \geq 0$ such that $P(0 \leq T \leq m) = \frac{1}{2}$)? (3p)

5. Let $K = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 1, \ 0 \leq y \leq 1 - x\}$ be a triangle in $\mathbb{R}^2$ and define the function $f(x, y) = x^2 + 2xy$. Calculate
$$\iint_K f(x, y) \, \mathrm{d}A. \tag{6p}$$