BK80A4010 Engineering Mechanics II

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Exam 4.5.2026

Allowed: Calculator (no restrictions). No written material besides the provided formulae sheet.

Exam has two sections:

- **Section A, which contains 10 short & easy questions each worth 1p**
- **Section B, which contains broader questions worth 40p in total (point values for each question in brackets)**

Student must get at least 6 points from section A, or section B won't be graded.

General advice:

- **Write your answers to all questions in the answer paper – not question paper!**
- **Answer section A on the front page of your answer paper**
    - **There is no NEED to provide explanations in section A - plain answers are sufficient (unless you think the question is a bit controversial; in this case please elaborate)**
    - **That said, calculations are naturally welcome**
    - **Please use CAPITAL LETTERS for the sake of clarity**
- **In section B answers, remember to document your solution – including intermediate steps and substitutions!**
- **Tasks don't need to be solved in order, but please mark clearly the task numbers**

Section A

1. Force $\boldsymbol{F} = 200\boldsymbol{i} + 50\boldsymbol{j} - 100\boldsymbol{k}$ acts in the direction of vector $\boldsymbol{r} = 4\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}$. What is the magnitude of this force? (Rounded to nearest integer.)

A) 150 N B) 180 N C) 229 N D) 1050 N

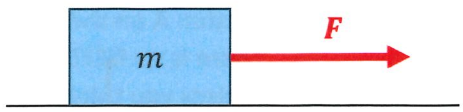
2. Force $\boldsymbol{F} = 60\boldsymbol{i} - 80\boldsymbol{j} + 50\boldsymbol{k}$ acts at point A = (1,4,5). What is the magnitude of moment that it creates subject to point B = (-3,2,4)? (Rounded to nearest integer.)

A) 425 B) 496 C) 586 D) 724

3. A ball-and-socket joint translates to how many (scalar) support reactions in FBD?

A) 1 B) 2 C) 3 D) 4

4. A box (m = 13 kg) is initially at rest, when a pulling force of $F = 55$ N is applied as pictured on the right. If the static and kinetic COFs are 0.4 and 0.28, respectively, what is the resulting acceleration of the box? (Two-decimal accuracy.)



A) $0.00\ m/s^2$ B) $0.31\ m/s^2$ C) $0.51\ m/s^2$ D) $1.48\ m/s^2$

5. We have derived a potential energy function $V(\theta)$ for a simple mechanism. When is the mechanism in equilibrium?

A) $\theta = 0$ B) $V(\theta) = 0$ C) $V'(\theta) = 0$ D) $V''(\theta) = 0$

6. A round metal bar (diameter d) that is attached to the wall is pulled by force P. The elastic modulus of this metal is 175 GPa, yield strength is 320 MPa and ultimate strength 490 MPa. As a result of the pulling, the bar experiences a strain of $450 \cdot 10^{-6}$. How does the metal behave – what region of the stress-strain curve are we in?

A) Elastic region B) Yielding region

C) Strain hardening region D) Can't say without knowing the values of P and d

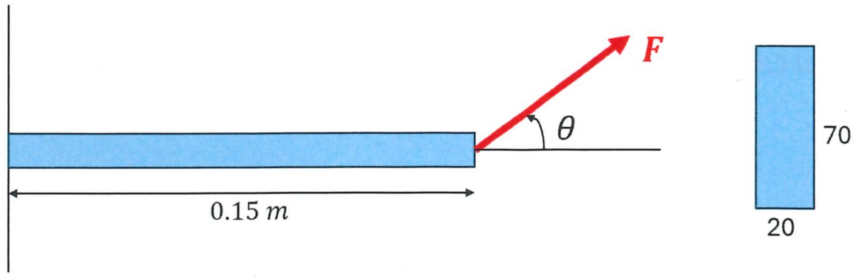
7. A circular shaft that has a diameter of 25 mm experiences a torque of 420 Nm. What is the greatest shear stress that is developed in the shaft? (Rounded to nearest integer.)

A) 0 MPa (no shear stress) B) 11 MPa C) 17 MPa D) 137 MPa

8. A short beam has been attached to the wall and is loaded by a force of F = 12 kN that acts at an angle of $\theta = 20°$ as shown below. The cross-section of the beam is shown on the right side (dimensions in millimeters). What is the greatest normal stress in the beam? (Accuracy of 3 significant digits.)

A) 8.06 MPa B) 37.7 MPa

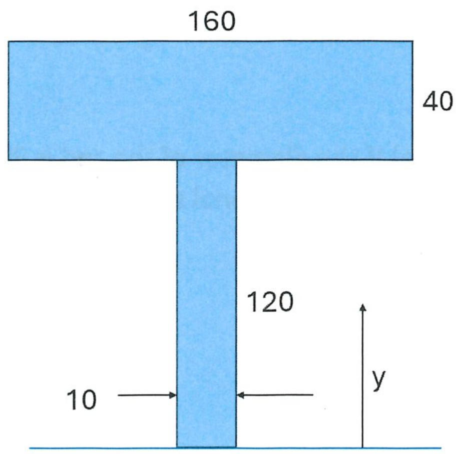
C) 45.7 MPa D) 110 MPa



9. What is the location of neutral axis $\bar{y}$ for the profile on the right? All dimensions are in millimeters. (Rounded to nearest integer.)

A) 80 mm B) 100 mm

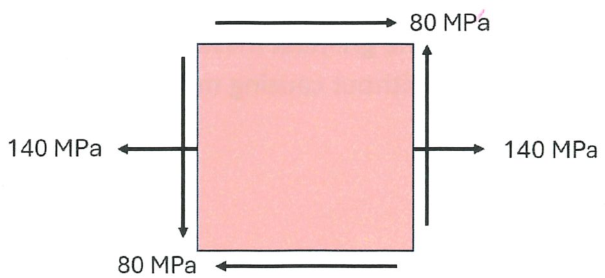
C) 112 mm D) 127 mm



10. The stress state of a certain point in our structure is pictured on the right. What is the Von Mises stress at this point? (Accuracy of 3 significant digits.)

A) 122 MPa B) 161 MPa

C) 197 MPa D) 394 MPa



SECTION B

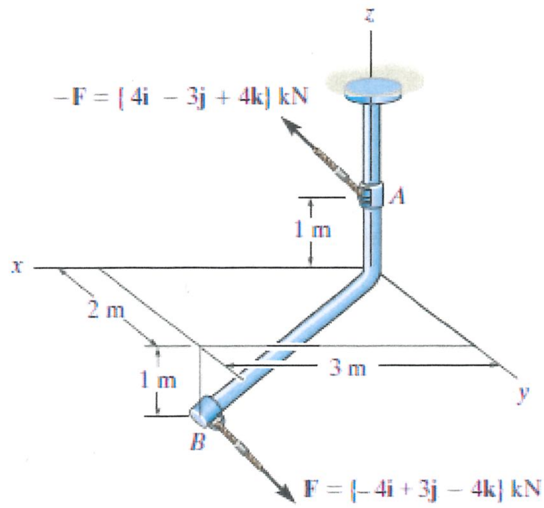
1. An eye bolt is in the origin of our coordinate system. Three forces act on the eye bolt: Force 1 has a magnitude of 600 N and it is directed towards point A = (2, -1, 5). Force 2 has a magnitude of 380 N and it is directed towards point B = (3, 7, -4). Force 3 has a magnitude of 220 N and it is directed towards point C = (1, 4, 8).

a) Calculate the forces 1-3 in vector form. Give the answers using 1-decimal accuracy. (4p)

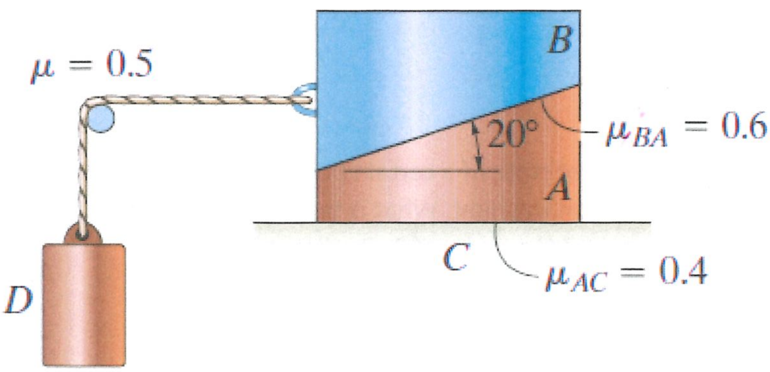
b) Calculate the magnitude of the external resultant force. (2p)

2. a) Calculate the moment of the couple acting on the rod. Give the answer in Cartesian vector form and calculate also the magnitude of the moment. (3p)

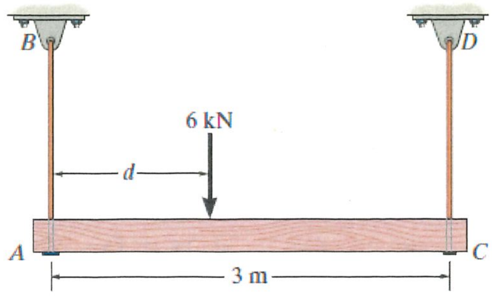
b) If the diameter of the rod is 90 mm, calculate the greatest torsional stress that is caused at point A. (3p)



3. Mass of block A is 50 kg and mass of block B is 30 kg. Using the given coefficients of static friction, determine the greatest mass of cylinder D without causing motion. (6p)



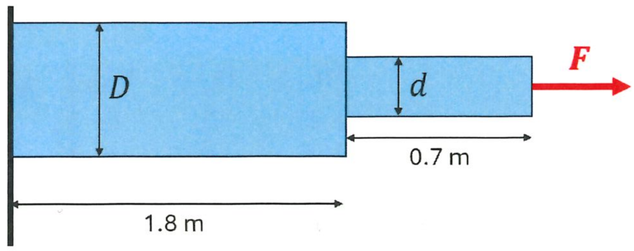
4. a) The beam is supported by two rods *AB* and *CD* that have cross-sectional areas of 12 mm^2 and 8 mm^2, respectively. Determine the position *d* of the 6-kN load so that the average normal stress in each rod is the same. (4p)



b) A 7500 W electric motor drives a driveshaft (diameter 35 mm, wall thickness 4 mm) at a speed of 300 rpm. What is the maximum shear stress in the driveshaft? (4p)

5. a) A cylindrical tank that has an external diameter of 1660 mm and a wall thickness of 18 mm is pressurized to 37 bar. Calculate the resulting Von Mises stress in the tank wall. (3p)

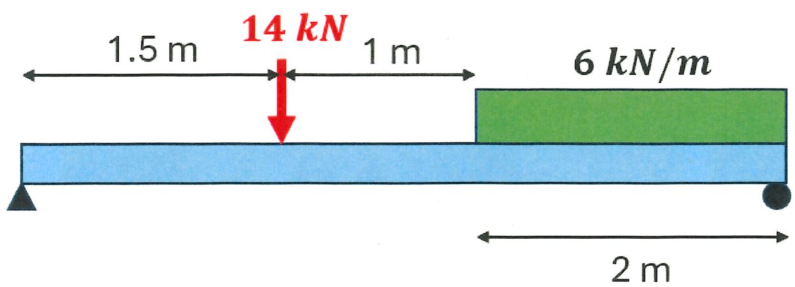
b) A circular bar shown on the right ($D = 40\ mm, d = 15\ mm$) has been attached to a wall and is then being pulled by a force of $F = 20\ kN$. Material is steel ($E = 210\ GPa, f_y = 355\ MPa$). Calculate the total elongation of the bar in millimeters. (3p)

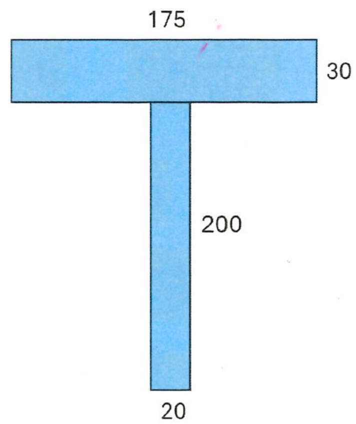


6. The T-beam (cross-section pictured below on the right; dimensions in mm) is subjected to the loading shown below on the left.

a) Calculate the greatest normal stress acting in the beam. (5p)

b) Calculate the greatest shear stress acting in the beam. (3p)





$\sum F_x = 0$ $\sum F_y = 0$ $\sum F_z = 0$ $F_i = F_i u_v = F_i \frac{v}{|v|}$ $\sum F_i = 0$

$v = (x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k$

$u_v = \frac{v}{|v|} = \frac{(x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$

$a \times b = \begin{vmatrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$

$F_i = F_i u_v = F_{i,x} i + F_{i,y} j + F_{i,z} k$

$M_O = r \times F = \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$

$\alpha_i = \cos^{-1}\left(\frac{F_{i,x}}{F_i}\right)$ $\beta_i = \cos^{-1}\left(\frac{F_{i,y}}{F_i}\right)$ $\gamma_i = \cos^{-1}\left(\frac{F_{i,z}}{F_i}\right)$

$a \times b = |a||b| \sin\theta\, n$ $M_{R_O} = \sum (r_i \times F_i)$ $M_a = u_a \cdot M_O$

$M_a = u_a \cdot (r \times F) = \begin{vmatrix} u_{a,x} & u_{a,y} & u_{a,z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$ $M_a = M_a u_a$ $\sum F = 0$ $\sum M_O = 0$

$F = F_x i + F_y j + F_z k$

$N = F_x$ $V = \sqrt{F_y^2 + F_z^2}$ $M = \sqrt{M_y^2 + M_z^2}$ $T = M_x$ $M = M_x i + M_y j + M_z k$

$\varphi_s = \tan^{-1}\mu_s$ $F_s = \mu_s N$ $F_k = \mu_k N$ $\theta = \tan^{-1}\left(\frac{l}{2\pi r}\right)$ $\varphi_s = \tan^{-1}\mu_s$

$M_u = Wr\tan(\theta + \varphi)$ $M_{d,s} = Wr\tan(\theta - \varphi)$ $M_{d,r} = Wr\tan(\varphi - \theta)$

$T_2 = T_1 e^{\mu\beta}$ $U_F = \int F\, ds$ $U_M = \int M\, d\theta$ $\delta U_F = F \cdot \delta s$ $\delta U_M = M \cdot \delta\theta$

$\Sigma\delta U = 0$ $T_1 + V_1 = T_2 + V_2$

$\frac{dV}{ds} = 0$ $\sigma = \frac{F}{A}$ $\varepsilon_{avg} = \frac{\Delta s' - \Delta s}{\Delta s}$ $\gamma = \frac{\pi}{2} - \theta'$ $\sigma = \frac{F}{A_0}$

$\varepsilon = \frac{\delta}{L_0}$ $\sigma_{true} = \sigma(1 + \varepsilon)$ $\varepsilon_{true} = \ln(1 + \varepsilon)$ $\varepsilon_f = \frac{L_f - L_0}{L_0}$

$\sigma = E\varepsilon$ $\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}$ $\tau = G\gamma$ $G = \frac{E}{2(1 + \nu)}$ $\delta = \frac{PL_0}{EA}$

$\delta = \int_0^L \frac{P(x)}{EA(x)} dx$ $\delta_T = \alpha \Delta T L_0$ $\sigma_T = \alpha \Delta T E$ $\sigma_{max} = K\sigma_{avg}$

$$\tau_{avg} = \frac{V}{A}$$ $$\tau_{max} = \frac{Tc}{J}$$ $$\tau = \frac{T\rho}{J}$$ $$J = \frac{\pi}{2}c^4$$ $$J = \frac{\pi}{2}(c_o^4 - c_i^4)$$ $$P = T\omega$$ $$\omega = 2\pi f$$

$$\phi = \int_0^L \frac{T(x)}{J(x)G}dx$$ $$\phi = \frac{TL}{JG}$$ $$\phi = \sum \frac{TL}{JG}$$ $$\tau_{avg} = \frac{T}{2tA_m}$$ $$\tau_{max} = K\frac{Tc}{J}$$

$$\sigma_{max} = \frac{Mc}{I}$$ $$\sigma = -\frac{My}{I}$$ $$\bar{y} = \frac{\int_A y\,dA}{\int_A dA}$$ $$\bar{y} = \frac{\sum \bar{y}_i A_i}{\sum A_i}$$ $$I_O = \frac{bh^3}{12}$$ $$I = I_O + Ad^2$$

$$I_O = \frac{\pi r^4}{4}$$ $$I_{tot} = I_1 + I_2 + \cdots + I_n$$ $$M_z = M\cos\theta$$ $$M_y = M\sin\theta$$

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$ $$\tan\alpha = \frac{I_z}{I_y}\cdot\tan\theta$$ $$\sigma_{max} = K\frac{Mc}{I}$$ $$\tau = \frac{VQ}{It}$$ $$Q = \sum y_i A_i$$

$$\tau_{max} = 1.5\frac{V}{A}$$ $$q = \frac{VQ}{I}$$ $$q = \tau t$$ $$\sigma_1 = \frac{pr}{t}$$ $$\sigma_2 = \frac{pr}{2t}$$

$$\sigma_a = \frac{P}{A}$$ $$\sigma_b = \frac{My}{I}$$ $$\sigma_{p1} = \frac{pr}{t}$$ $$\sigma_{p2} = \frac{pr}{2t}$$ $$\sigma_x = \sigma_{a,x} + \sigma_{b,x} + \sigma_{p,x}$$ $$\sigma_y = \sigma_{a,y} + \sigma_{b,y} + \sigma_{p,y}$$

$$\tau_s = \frac{VQ}{It}$$ $$\tau_t = \frac{T\rho}{J}$$ $$\tau_{xy} = \tau_s + \tau_t$$ $$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tan 2\theta_s = \frac{\sigma_y - \sigma_x}{2\tau_{xy}}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\tau_{max,abs} = \max\left\{\left|\frac{\sigma_1}{2}\right|, \left|\frac{\sigma_2}{2}\right|, \left|\frac{\sigma_1 - \sigma_2}{2}\right|\right\}$$

$$\tau_{max,in-plane} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$ $$n = \frac{\sigma_{fail}}{\sigma_{eq}}$$ $$\sigma_{allow} = \frac{\sigma_{fail}}{n}$$ $$\sigma_{eq} \le \sigma_{allow}$$

$$\sigma_{VM} = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$$
$$Yields, if\ \sigma_{VM} > f_y$$

$$\sigma_T = \max\{|\sigma_1|, |\sigma_2|, |\sigma_1 - \sigma_2|\}$$
$$Yields, if\ \sigma_T > f_y$$

$$\sigma_{MNS} = \max\{|\sigma_1|, |\sigma_2|\}$$
$$Fails, if\ \sigma_{MNS} > f_u$$