BK80A2520 Advanced finite element methods in solid and structural mechanics

Exam – 29 April 2026

- A graphical calculator is allowed.
- Note that Tasks 1 and 15 are hand calculation tasks. Write intermediate steps and final results clearly.
- In multiple-choice questions, each correct answer is worth one point. Select only the statements that are correct.
- In multiple-choice questions, each incorrect answer deducts one point within that task. The score for a task cannot drop below zero. The number of correct answers varies from task to task.
- The exam is graded on a 60-point scale. The total points may be scaled if necessary.
- Write all answers on the answer sheet. Answers written on the question paper will not be graded.

1. The motion of a body in 2D is given by the position field

$$\boldsymbol{x}(\boldsymbol{X}) = \begin{bmatrix} X + 0.2XY \\ Y - 0.3X^2 \end{bmatrix},$$

Consider the material point in the initial configuration

$$\boldsymbol{X}_p = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

(a) (1 p) Compute the current position $\boldsymbol{x}$ of this material point, i.e. $\boldsymbol{x}(\boldsymbol{X}_p)$.

(b) (2 p) Compute the deformation gradient

$$\boldsymbol{F} = \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{X}}, \qquad F_{ij} = \frac{\partial x_i}{\partial X_j},$$

and evaluate it at the given material point $\boldsymbol{X}_p$. Present the derivatives of the components and evaluate them at this point.

(c) (2 p) Compute the determinant of the deformation gradient $J = \det(\boldsymbol{F})$ and briefly state whether the deformation locally increases or decreases the area at the material point $\boldsymbol{X}_p$.

(d) (2 p) Compute the right Cauchy–Green deformation tensor at the given material point $\boldsymbol{X}_p$

$$\boldsymbol{C} = \boldsymbol{F}^T \boldsymbol{F}.$$

(e) (2 p) Compute the Green–Lagrange strain tensor at the given material point $\boldsymbol{X}_p$

$$\boldsymbol{E} = \frac{1}{2}(\boldsymbol{C} - \boldsymbol{I}).$$

What can you say about the strains?

(f) (2 p) Compute the displacement gradient $\nabla \boldsymbol{u}$ at the given material point $\boldsymbol{X}_p$. The position field can be written as

$$\boldsymbol{x} = \boldsymbol{X} + \boldsymbol{u}.$$

Thus, in this case, the displacement field is

$$\boldsymbol{u}(\boldsymbol{X}) = \begin{bmatrix} 0.2XY \\ -0.3X^2 \end{bmatrix},$$

and

$$\nabla \boldsymbol{u} = \frac{\partial \boldsymbol{u}}{\partial \boldsymbol{X}}, \qquad \text{or in index notation} \qquad (\nabla \boldsymbol{u})_{ij} = \frac{\partial u_i}{\partial X_j}.$$

5. In nonlinear finite element analysis,
    A. the equilibrium equations are usually solved iteratively.
    B. the tangent stiffness matrix may depend on the current displacement state.
    C. the stiffness matrix is always constant during the whole analysis.
    D. Newton's method is commonly used to solve the nonlinear equilibrium equations.
    E. the residual force vector is always zero at the beginning of the first iteration.
    F. geometric nonlinearities, material nonlinearities, and contact can lead to nonlinear finite element problems.
    G. nonlinear finite element analysis is always more accurate than linear finite element analysis.
    H. incremental load stepping (sub loads) is often used to improve the likelihood of finding a converged solution in nonlinear problems.
    I. nonlinear problems can always be solved by a single linear equation solve.
    J. convergence can be checked using residual norms and/or displacement increment norms.

6. In a 2D steady-state heat conduction finite element model with isotropic conductivity property k, what statements about the element conductivity matrix $\boldsymbol{k}^e$ are always correct?
    A. it stores thermal expansion coefficients for the element
    B. it relates nodal temperatures to nodal heat flows within the element
    C. it relates nodal displacements to nodal forces within the element.
    D. it is a diagonal matrix
    E. it is a symmetric square matrix
    F. it enforces essential boundary conditions within the element
    G. it is a 2×2 matrix

7. Which statements are correct about using our Matlab implementation and Ansys for thermoelastic analysis?
    A. Ansys automates meshing allowing to consider computational domains of complex geometry with less implementation effort
    B. Ansys uses different physical model for thermal expansion
    C. Ansys does not allow applying different temperatures at different nodes
    D. Unlike our Matlab implementation, Ansys allows bi-direction coupling between thermal and static analysis
    E. Unlike our Matlab implementation, Ansys allows using also third-order elements for thermoelastic analysis (elements with cubic interpolation for unknown variables within the element)
    F. While we used finite element method for solving both thermal conduction and deformation problems, Ansys uses central difference method for solving thermal conduction problems

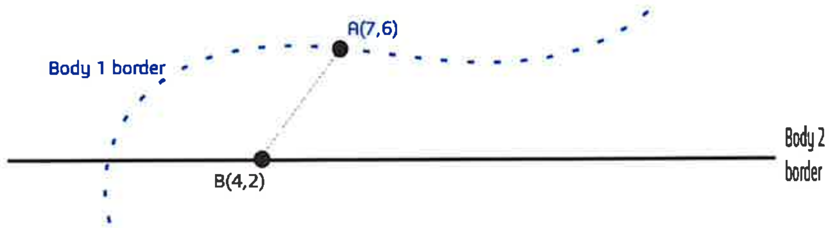
11. In penalty contact approaches, if the penalty factor p_n is chosen very large, the solution typically leads to:
   A. Improved convergence, but longer computation;
   B. No effect on the global system behavior;
   C. Convergence difficulties;
   D. Exact enforcement of contact without penetration;

12. In penalty contact approaches, the increasing penalty factor p_n in two times leads to the following, if the surface contact area A:
   A. Increasing penetration approximately by two times;
   B. Decreasing penetration approximately by two times
   C. Depends on the surface contact, increasing by $2 \times A$;
   D. Any of the proposed answers is incorrect

13. In the Lagrange multiplier contact approach, what happens to penetration between contacting bodies?
   A. Small penetration is allowed, depending on a parameter;
   B. No penetration occurs, and the contact is enforced exactly;
   C. Penetration increases with contact area;
   D. Penetration depends only on material stiffness;

14. In the augmented Lagrangian based contact approach, how is the contact condition enforced?
   A. Only by using a penalty parameter;
   B. Only by using Lagrange multipliers;
   C. By combining Lagrange multipliers with a penalty term;
   D. By increasing material stiffness;

15. There is an interaction of two bodies (on the pic dimensions is given in mm), the penalty approach with the penalty factor $p_n = 10^6 \frac{N}{mm}$ is used to reinforce the contact. What would be the contact force between points A and B (3 points)?



Points distribution:

1. Knowing that, it depends on the projection - 1 point
2. Calculating projection distance (4 mm) - 1 point
3. Calculating force: $\delta \times p_n = 4mm \times 10^6 \dfrac{N}{mm} = 4 \times 10^6 N$- 1 point

21. Linear buckling concept

Which statements are correct?

A. Buckling refers to a sudden change in deformation pattern under compressive loading.

B. The critical buckling load is the load at which the structure becomes unstable.

C. When the load exceeds the critical value, the structure may experience large out-of-plane deformations.

D. Buckling occurs only after material yielding.

E. The critical buckling load corresponds to maximum stress in the material.

F. After buckling, the structure always collapses immediately.

22. Buckling of composite plates

A rectangular composite plate is compressed along one direction. Which statements are correct?

A. Increasing the plate thickness increases the buckling resistance.

B. Fiber orientation significantly affects the buckling load.

C. Material stiffness in the loading direction affects the buckling load.

D. Buckling load is independent of plate geometry.

E. Fiber orientation has no influence on buckling behavior.

F. Increasing thickness reduces the buckling resistance.