

BM40A0102 Foundations of Information Processing

Exam 2

No written material is allowed in the exam.

No electronic devices are allowed (no calculators, phones, laptops, tablets, etc.).

The students in the international program must reply in English.

The students in the Finnish-speaking program can reply in English or in Finnish.

Please remember to explain your answers carefully in details.

1. Concepts (10 p):

- (a) Merge sort algorithm: illustrate by drawing step-by-step how the algorithm sorts the following numbers 12, 7, 22, 27, 32, 8, 6, 4 in the increasing order.
- (b) What is the difference between function modules and procedure modules?
- (c) Show by drawing the difference between the breadth-first and depth-first search.
- (d) Complexities:
 - i. Define the polynomial complexity and the exponential complexity as mathematical formulas as a function of n .
 - ii. Show the difference of these complexities by drawing the corresponding functions.
- (e) Data structures:
 - i. Illustrate by drawing what is the difference between graphs and trees.
 - ii. Can a tree be a part of a graph? Explain your answer.

2. Algorithms (10 p):

- (a) Is the algorithm recursive or iterative?
Tell what parts of the algorithm show this. Justify your answer carefully. It is not enough to just tell that the algorithm is iterative since it is not recursive, or vice versa.

```
MODULE factorial(n) RETURNS n!  
  IF n = 0 THEN  
    RETURN 1  
  ELSE  
    RETURN n * factorial(n - 1)  
  ENDIF  
ENDMODULE
```

- (b) Is the algorithm recursive or iterative?
Tell what parts of the algorithm show this. Justify your answer carefully. It is not enough to just tell that the algorithm is iterative since it is not recursive, or vice versa.

```
MODULE algorithm(n) RETURNS n!  
  k:=1  
  WHILE n>1 DO  
    k:=k*n  
    n:=n-1  
  ENDWHILE  
  RETURN k  
ENDMODULE
```

- (c) Show all the calls of the function `factorial` and the corresponding values returned by the function when it is called as `factorial(4)`. For example, `factorial(0)` returns 1 directly, but `factorial(1)` does not give the value directly. Explain how the values are formed.

Note: remember also to present how to return from `factorial(0)` back to the final result.

Justify your answer carefully.

3. Data encoding (10 p):

Let us calculate the numbers represented in *the two's complement* notation as follows

$$\begin{array}{r|l|l} 0011_2 & 1101_2 & 0111_2 \\ + 0010_2 & + 1110_2 & + 1011_2 \\ \hline \end{array}$$

keeping them in the binary representation during computations.

- (a) **Compulsory subtask:** must be done successfully to receive points from Question 3.
- Show the full range of all values represented in the two's complement in case of the bit patterns of length **four** as follows (so complete the following representation from the maximum value to the minimum value):

0111	7
⋮	
0000	0
1111	-1
⋮	
1000	-8

- (b) Calculate the given computations in the two's complement notation, using the range of values defined and showing your binary calculations.
Explain bit by bit how the addition happens.
- (c) Double check the results by changing the original numbers to the decimal system and by performing the same computations.
- (d) Show the full range of all values represented in the two's complement in case of the bit patterns of length **three** (instead of four).

4. Data compression using the Huffman coding (10 p):

Let us assume that there are the estimated probabilities of occurrence of the symbols $p(x_i)$ as follows:

A	0.40
E	0.20
H	0.15
O	0.15
L	0.10

- (a) **Compulsory subtask:** must be done successfully to receive points from Question 4.
- Why does the Huffman coding compress the representation of data?
Justify the benefit of the method by explaining why the method compresses.
- Note: It is not enough just to reply “it compresses” or “it is based on probabilities” or “two smallest probabilities are combined”.

- (b) **Compulsory subtask:** must be done successfully to receive points from Question 4.
- What would be a case where the Huffman coding cannot compress?
Explain why as based on to the idea of the method.
Note: It is not enough just to reply “all probabilities are same”.
- (c) Apply the Huffman coding to generate the corresponding Huffman codes for each symbol. Tell step by step how your coding works.
- (d) How many bits per symbol are needed on the average using your coding?
Remember to show how you compute this average.
- (e) How many bits per symbol are needed on the average if there is no compression (so 5 symbols are represented directly without any coding for compression)? Justify your reply and compare to the result of the Huffman coding.
- (f) Show whether you can decode or cannot decode the following based on your coding, and tell what string is the result of decoding if you can decode:
- i. 10011
 - ii. 0011110

5. **Logic and reasoning** (10 p):

Prove using the semantic method (the truth table) that r can be concluded from the following premises:

$$p \wedge q \rightarrow r$$

$$q \rightarrow p$$

$$q$$

- (a) **Compulsory subtask:** must be done successfully to receive points from Question 5.
- Which ones of the following propositions are correct and which ones incorrect for proving?:
- i. $(p \wedge q \rightarrow r) \wedge (q \rightarrow p) \wedge q$
 - ii. $((p \wedge q \rightarrow r) \wedge (q \rightarrow p) \wedge q) \rightarrow r$
 - iii. $((p \wedge q \rightarrow r) \wedge (q \rightarrow p)) \rightarrow r$
- Please explain carefully why. Without proper explanations no points from Question 5 are given.
- (b) **Compulsory subtask:** must be done successfully to receive points from Question 5.
- How is the selected proposition used for proving?
Do not yet present the truth table here, but tell what condition the proposition must fulfill.
- (c) The truth table:
- i. Present the truth table.
 - ii. Explain what and how can be concluded from the truth table.
Show the corresponding **place** in the truth table.
- (d) The complexity:
- i. How many the atomic propositions are in the truth table? Show them in the truth table.
 - ii. How many rows are in the truth table? Show the rows in the truth table.
 - iii. How are the number of the rows in the truth table and the number of the atomic propositions related to each other?
 - iv. Define the number of the rows as a function of the number of the atomic propositions. Explain your answer.
 - v. How many rows would be in the truth table if there was one atomic proposition more than now?

Please remember to explain your answers carefully in details.