# BM20A7901 Differential Calculus
# Exam 23.3.2026

Only the following written materials are allowed in the exam: MAOL, BETA, Tekniikan taulukkokirja (Valtanen), Matematiikan taulukkokirja (Valtanen), and a handwritten A4-sized formula sheet.

Calculators are not allowed.

Answer each question.

1. Determine the global minimum and maximum values of the function $f\colon \mathbb{R} \to \mathbb{R}$,
$$f(x) = \frac{2x}{x^2+1}.$$
Also, state the values of $x$ at which the function attains its global minimum and maximum. (6p)

2. Let $f(x) = e^{-2x}$.
   (a) Form the 1st, 2nd, and 3rd degree Taylor polynomials for $f(x)$ about $x = 0$. (3p)
   (b) The error formula of the $n^{\text{th}}$ degree Taylor polynomial $P_n(x)$ developed about the origin is given by
$$f(x) - P_n(x) = \frac{f^{(n+1)}(c_x)}{(n+1)!}x^{n+1}, \quad \text{where } x \in \mathbb{R} \text{ and } c_x \in (\min\{0,x\}, \max\{0,x\}).$$
   Consider the value of the 3rd degree Taylor polynomial you obtained in part (a) at $x = 1$. Show that
$$|f(1) - P_3(1)| \leq \frac{2}{3}.$$ (3p)

3. (a) Find the equation of the tangent line to the hyperbola
$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$
passing through the point $(\sqrt{5}, \frac{3}{2})$. (3p)
   (b) Let $f(x,y) = y\cos(x)$. Determine the equation of the tangent plane to the graph of $f$ at $(0,0,f(0,0))$. (3p)

4. Using the method of Lagrange multipliers, determine the largest and smallest value of the function
$$f(x,y) = xy$$
in the set $\{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$. Also, state the corresponding points $(x,y)$ where these extreme values are attained. (6p)

5. Using the method of least squares, find the line of best fit $y = ax + b$ for the data points
$$(0,0),\ (1,1),\ (1,2).$$
Here, "best fit" is understood to mean that the line minimizes the sum of the squares of the residuals. (6p)