BK80A4000 Engineering Mechanics I / Olli-Pekka Hämäläinen

Exam 9.3.2026

Allowed: Calculator (no restrictions). No written material besides the provided formulae sheet.

Exam has two sections:

- **Section A, which contains 10 short & easy questions each worth 1p**
- **Section B, which contains broader questions worth 40p in total (point values for each question in parentheses)**

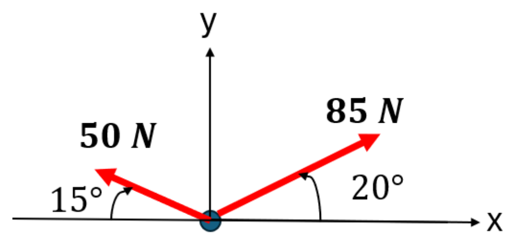
Student must get at least 6 points from section A, or section B won't be graded.

General advice:

- **Write your answers to all questions in the answer paper – not question paper!**
- **Answer section A on the front page of your answer paper**
  - **There is no need to provide explanations in section A - plain answers are sufficient (unless you think the question is a bit controversial; in this case please elaborate)**
  - **Please use CAPITAL LETTERS for the sake of clarity**
- **Tasks don't need to be solved in order, but please mark clearly the task numbers**

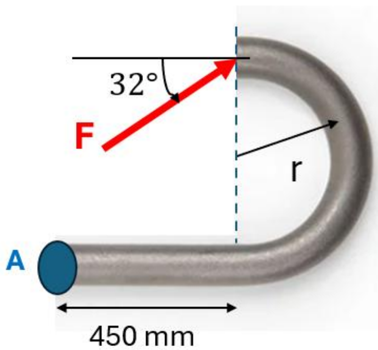
Section A

1. What is the magnitude of the resultant force acting on the particle on the right?



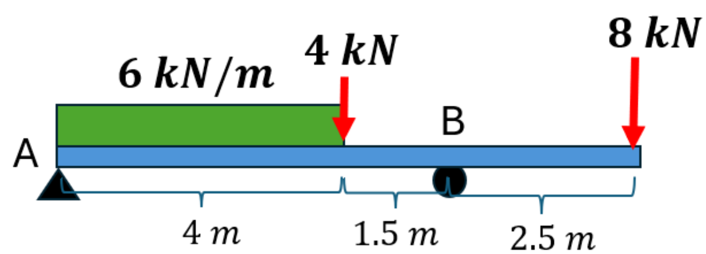
A) 53 N B) 74 N C) 129 N D) 135 N

2. Force F = 600 N and the radius of the hook is r = 300 mm. What is the magnitude of moment that this generates at A?



A) 38 Nm B) 162 Nm C) 238 Nm D) 448 Nm

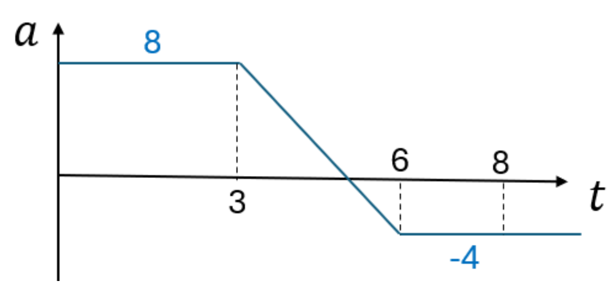
3. What is the support reaction at B?



A) 12.7 kN B) 13.5 kN

C) 23.3 kN D) 28.3 kN

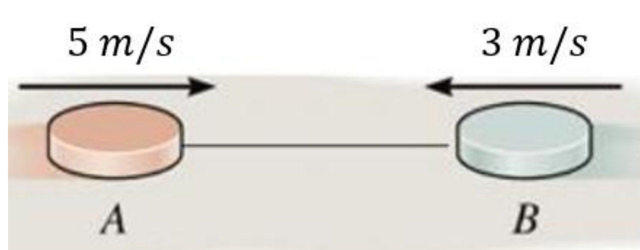
4. On the right you can see an a-t-graph that depicts the motion of a car (SI units). What is the speed of the car at t = 8 s?



A) 0 m/s B) 10 m/s

C) 22 m/s D) 42 m/s

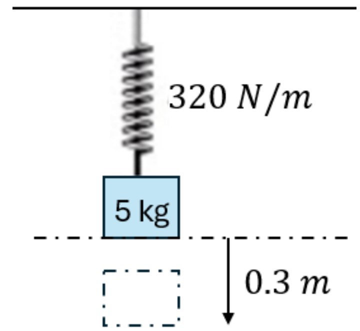
5. Mass of disk A is 2 kg and mass of disk B is 1 kg. The disks slide along a frictionless plane and collide head-on. If the coefficient of restitution is 0.5, what is the speed of disk A after the hit?



A) 1 m/s B) 3 m/s

C) 3.5 m/s D) 4 m/s

6. A block hanging on a spring is in equilibrium in the picture on the right. Then it is displaced 0.3 m downwards from its equilibrium position and let loose to vibrate. What is the differential equation of this vibration?



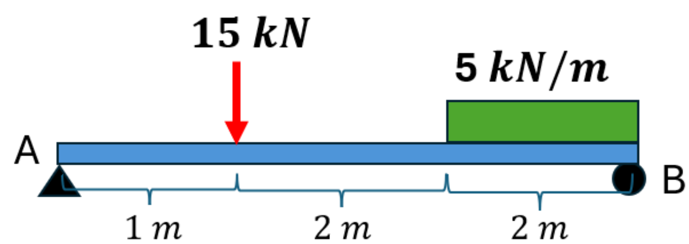
A) $5\ddot{x} + 320x = 0$ B) $5\ddot{x} + 64\dot{x} + 320x = 0$

C) $5\ddot{x} + 320x = 0.3\sin(8t)$ D) $5\ddot{x} + 320x = 96\sin(8t)$

7. Members of a simple truss are only capable of carrying… what kind of forces?

A) Axial forces   B) Gravitational forces   C) Shear forces   D) Tangential forces

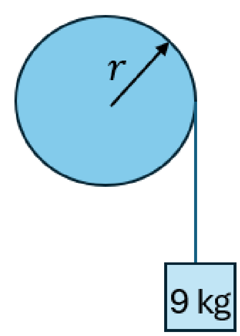
8. What is the greatest absolute internal bending moment ($|M_{max}|$) in the beam on the right?



A) 14 kNm   B) 25 kNm

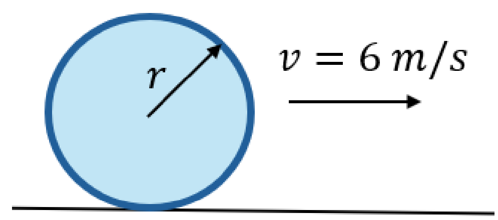
C) 55 kNm   D) 70 kNm

9. A rope has been wrapped around a disk (r = 0.4 m, mass 60 kg). The disk is pinned at its center point so that it remains in place. Then a block ($m = 9\ kg$) is attached to the free end of the rope and let loose. What is the resulting angular acceleration of the disk?



A) 3.7 rad/s^2 B) 5.7 rad/s^2 C) 7.4 rad/s^2 D) 24.5 rad/s^2

10. A hollow ball (m = 5 kg) is rolling without slipping along a horizontal plane with a linear velocity of 6 m/s. What is the kinetic energy of the ball?

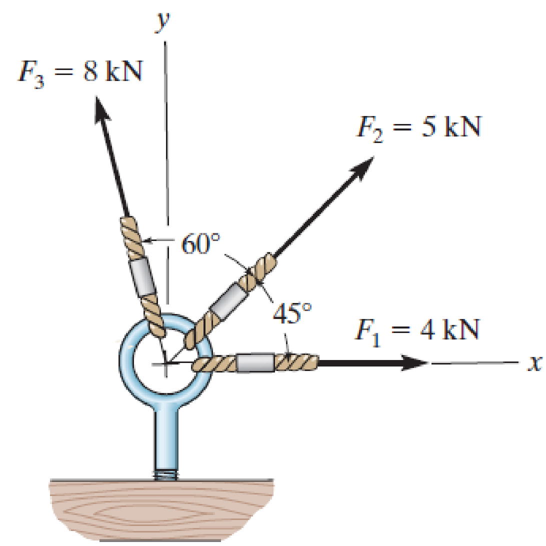


A) 60 J   B) 90 J   C) 150 J   D) 210 J

(See next page for Section B)

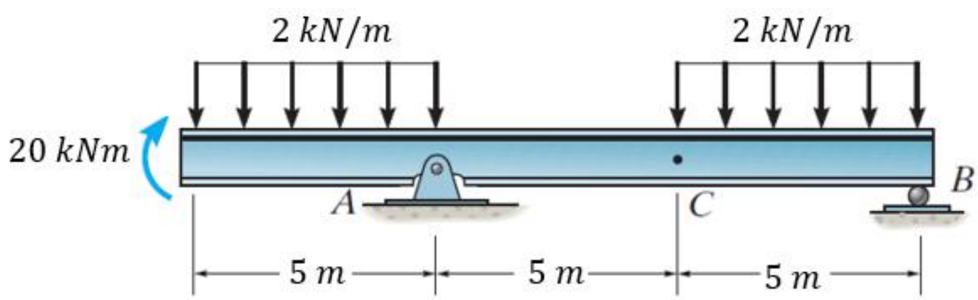
Section B

1. Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis. (5p)



2. a) Draw a FBD and calculate the support reactions for the beam shown below. (3p)

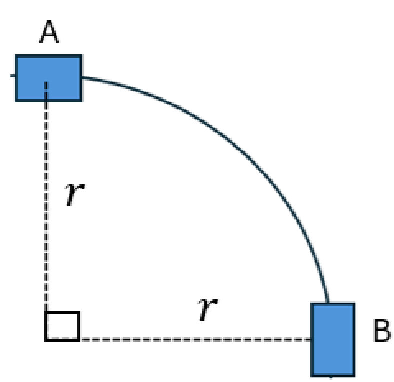
b) Sketch the shear and moment diagrams for the beam. (4p)



3. A car (mass 1700 kg) starts from rest at point A and starts to accelerate on a flat road with acceleration of $a(s) = 0.2s$ along the path shown. Radius is r = 30 m. (Picture from above.)

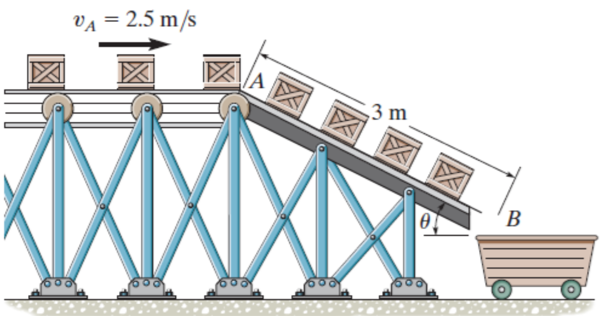
a) What is the speed and total acceleration of the car at B? (5p)

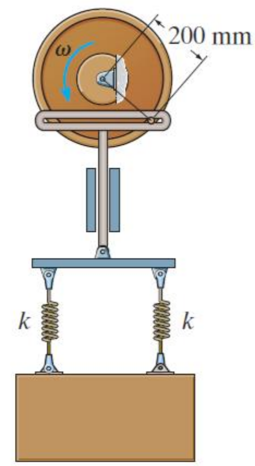
b) What is the magnitude of the friction force that is trying to keep the car on the road at B? (2p)



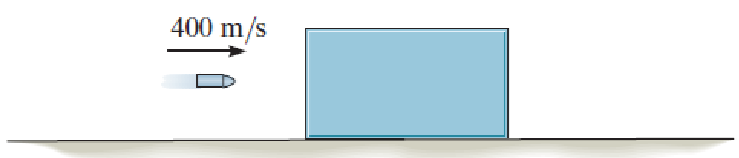
4. a) The conveyor belt delivers each 12-kg crate to the ramp at A such that the crate's initial speed is $v_A = 2.5\ m/s$, directed down along the ramp. If the ramp has an incline of $\theta = 18°$, calculate what is the greatest coefficient of kinetic friction μ_k so that the crates will still slide off at B and fall into the cart. (Illustration below on the left.) (5p)

b) The spring system is connected to a crosshead that oscillates vertically when the wheel rotates with a constant angular velocity of $\omega = 6\ rad/s$. The springs each have a stiffness of $k = 2500\ N/m$. The block (underneath the springs) has a mass of 50 kg. What is the amplitude of the resulting steady-state vibration? Ignore the mass of other parts of the system. (Illustration below on the right.) (4p)

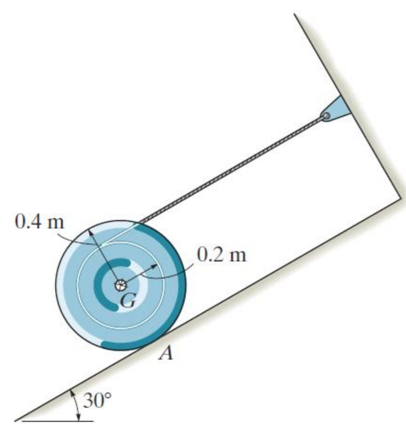




5. The 20-g bullet is traveling at 400 m/s when it becomes embedded in the 2-kg stationary block. Determine the time and distance that it takes for the block to slide before it stops. The coefficient of kinetic friction between the block and the plane is $\mu_k = 0.2$. (6p)



6. The 100-kg spool is kept at rest on the inclined surface for which the coefficient of kinetic friction is $\mu_k = 0.1$. Determine the angular velocity of the spool when $t = 4\ s$ after it is released. (The rope stays connected to the wall; rope just unwinds from the reel.) The radius of gyration about the mass center is $k_G = 0.25\ m$. (6p)



BK80A4000 Engineering Mechanics I / Formulae sheet

$$\sum F = ma \qquad F_R = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2} \qquad \theta_R = tan^{-1}\left(\frac{\sum F_y}{\sum F_x}\right) \qquad M = Fd$$

Distributed loadings: Uniform $F_R = wL, d = L/2$

Triangular $F_R = \frac{wL}{2}, d = L/3$

Trapezoidal $F_R = \frac{L(w_A+w_B)}{2}, d = \frac{L(w_A+2w_B)}{3(w_A+w_B)}$

$$v = \frac{ds}{dt} \quad a = \frac{dv}{dt} = \frac{d^2s}{dt^2} \quad a\,ds = v\,dv \quad v = \sqrt{v_x^2 + v_y^2} \quad a = \sqrt{a_x^2 + a_y^2} \quad \theta_v = \tan^{-1}\left(\frac{v_y}{v_x}\right) \quad \theta_a = \tan^{-1}\left(\frac{a_y}{a_x}\right)$$

$$s = s_0 + v_0 t + \frac{1}{2}a_c t^2 \quad v = v_0 + a_c t \quad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$a = \sqrt{a_t^2 + a_n^2} \quad a_t = \frac{dv}{dt} \quad a_n = \frac{v^2}{\rho} \quad \rho = \frac{[1 + (f'(x))^2]^{\frac{3}{2}}}{|f''(x)|} \quad \omega = \dot{\theta} = \frac{d\theta}{dt} \quad \alpha = \ddot{\theta} = \frac{d^2\theta}{dt^2} \quad \sum F_r = ma_r$$

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{\dot{r}^2 + (r\omega)^2} \quad a = \sqrt{a_r^2 + a_\theta^2} \quad a_r = \ddot{r} - r\omega^2 \quad a_\theta = r\alpha + 2\dot{r}\omega \quad \sum F_\theta = ma_\theta$$

$$\sum F = ma \quad F = G\frac{m_1 m_2}{r^2} \quad F_\mu = \mu N \quad \sum F_x = ma_x \quad \sum F_y = ma_y \quad \sum F_t = ma_t \quad \sum F_n = ma_n$$

$$\psi = \tan^{-1}\left(\frac{r}{\left(\frac{dr}{d\theta}\right)}\right) \quad \psi = \tan^{-1}\left(r\frac{d\theta}{dr}\right)$$

$$U_{1-2} = \int_{s_1}^{s_2} F\cos\theta\,ds \quad U_{1-2} = -\left(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2\right) \quad T_1 + \sum U_{1-2} = T_2 \quad \sum T_1 + \sum U_{1-2} = \sum T_2$$

$$P = \frac{dU}{dt} \quad P = Fv \quad \epsilon = \frac{Power\ output}{Power\ input} \quad \sum T_1 + \sum V_1 = \sum T_2 + \sum V_2 \quad L = mv$$

$$mv_1 + \sum\int_{t_1}^{t_2} F\,dt = mv_2 \quad L_1 + \sum I = L_2 \quad \sum m_i v_{i,1} + \sum\int_{t_1}^{t_2} F_i\,dt = \sum m_i v_{i,2}$$

$$m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2 \quad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1} \quad (H_O)_z = (d)(mv) \quad \sum \mathbf{M}_O = \dot{\mathbf{H}}_O$$

$$(\mathbf{H}_O)_1 + \sum\int_{t_1}^{t_2} \mathbf{M}_O dt = (\mathbf{H}_O)_2 \quad x(t) = A\sin(pt) + B\cos(pt) \quad x(t) = C\sin(pt + \varphi) \quad C = \sqrt{A^2 + B^2}$$

$$\ddot{x} + p^2 x = 0 \quad p = \sqrt{\frac{k}{m}} \quad \tau = \frac{2\pi}{p} \quad f = \frac{1}{\tau} = \frac{p}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$m\ddot{x} + c\dot{x} + kx = 0$$ $$c_c = 2mp$$ $$x = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$$ $$x = (A + Bt)e^{-pt}$$

$$\lambda_1 = -\frac{c}{2m} + \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$
$$\lambda_2 = -\frac{c}{2m} - \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

$$x = D\left[e^{-\left(\frac{c}{2m}\right)t}\sin(p_d t + \phi)\right],$$ $$p_d = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2} = p\sqrt{1 - \left(\frac{c}{c_c}\right)^2},$$ $$\tau_d = \frac{2\pi}{p_d}.$$

$$\ddot{x} + p^2 x = \frac{F_0}{m}\sin(\omega t)$$ $$x = x_c + x_p = A\sin pt + B\cos pt + \frac{\frac{F_0}{k}}{1 - \left(\frac{\omega}{p}\right)^2}\sin\omega t.$$ $$x_p = \frac{\frac{F_0}{k}}{1 - \left(\frac{\omega}{p}\right)^2}\sin\omega t$$

$$m\ddot{x} + c\dot{x} + kx = F_0\sin\omega t.$$ $$x(t) = x_c + x_p$$ $$x_p = C'\sin(\omega t + \phi')$$

$$C' = \frac{\frac{F_0}{k}}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left[2\left(\frac{c}{c_c}\right)\left(\frac{\omega}{p}\right)\right]^2}}$$ $$\phi' = \tan^{-1}\left[\frac{2\left(\frac{c}{c_c}\right)\left(\frac{\omega}{p}\right)}{1 - \left(\frac{\omega}{p}\right)^2}\right]$$

$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum M_O = 0$$

$$\sum F_x = 0, \quad \sum F_y = 0$$

$$\frac{dV}{dx} = -w(x)$$ $$\frac{dM}{dx} = V(x)$$

$$r_B = r_A + r_{B/A}$$ $$v_B = v_A$$ $$a_B = a_A$$ $$\omega = \frac{d\theta}{dt}$$ $$\alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt}$$ $$v = \omega r$$ $$a_t = \alpha r$$

$$\alpha\, d\theta = \omega\, d\omega$$ $$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$$ $$\omega = \omega_0 + \alpha_c t$$ $$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

$$a_n = \frac{v^2}{r} = \frac{(\omega r)^2}{r} = \omega^2 r$$ $$a = \sqrt{a_t^2 + a_n^2}$$ $$r_B = r_A + r_{B/A}$$ $$v_B = v_A + v_{B/A}$$

$$v_{B/A} = \omega r_{B/A}$$ $$v_{B/A,x} = -\omega r_{B/A,y}$$ $$v_{B/A,y} = \omega r_{B/A,x}$$

$$a_B = a_A + (a_{B/A})_t + (a_{B/A})_n$$ $$(a_{B/A})_t = \alpha r_{B/A}$$ $$(a_{B/A})_n = \omega^2 r_{B/A}$$

Body shape	Moment of inertia
Particle on string (length r)	mr^2
Disk (radius r)	$\frac{1}{2}mr^2$
Bar attached at midpoint (length l)	$\frac{1}{12}ml^2$
Bar attached at end point (length l)	$\frac{1}{3}ml^2$
Solid ball (radius r)	$\frac{2}{5}mr^2$
Hollow ball (radius r)	$\frac{2}{3}mr^2$
Any shape (radius of gyration r_G)	mr_G^2

$$\boldsymbol{M = I\alpha}$$

$$\boldsymbol{I = \int_m r^2\, dm}$$

$$\begin{cases} \sum F_x = m(a_G)_x \\ \sum F_y = m(a_G)_y \\ \sum M_G = 0 \end{cases}$$

$$\begin{cases} \sum F_n = m(a_G)_n = m\omega^2\rho \\ \sum F_t = m(a_G)_t = m\alpha\rho \\ \sum M_G = 0 \end{cases}$$

$$\begin{cases} \sum F_n = m(a_G)_n = m\omega^2 r_G \\ \sum F_t = m(a_G)_t = m\alpha r_G \\ \sum M_G = I_G\alpha \end{cases}$$

$$\begin{cases} \sum F_x = m(a_G)_x \\ \sum F_y = m(a_G)_y \\ \sum M_G = I_G\alpha \end{cases}$$

$$\boldsymbol{a_G = \alpha r}$$

$$\boldsymbol{F_\mu = \mu_k N}$$

$$\boldsymbol{\sum M_P = \sum (\mathcal{M}_k)_P}$$

$$\boldsymbol{T = \frac{1}{2}mv_G^2}$$

$$\boldsymbol{T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2}$$

$$\boldsymbol{T = \frac{1}{2}I_O\omega^2}$$

$$\boldsymbol{U_F = \int F\, ds}$$

$$\boldsymbol{U_M = \int M\, d\theta}$$

$$\boldsymbol{L = mv_G}$$

$$\boldsymbol{H = I_G\omega}$$

$$\boldsymbol{L = mv_G}$$
$$\boldsymbol{H_G = I_G\omega}$$

$$\boldsymbol{H_O = I_O\omega}$$

$$\boldsymbol{I_O = I_G + mr_G^2}$$

$$\boldsymbol{H_O = I_G\omega + dmv_G}$$

$$\boldsymbol{m(v_G)_1 + \sum \int_{t_1}^{t_2} F\, dt = m(v_G)_2}$$

$$\boldsymbol{I_G\omega_1 + \sum \int_{t_1}^{t_2} M_G\, dt = I_G\omega_2}$$